Electron Transport Through Two Branch Interferometer with Rashba Spin Orbit Interaction

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Abstract

In this paper, a theoretical model for electron transport through two branch interferometer with one quantum dot in each of its arms has been considered. Both a magnetic flux $\Phi$ applied on the circuit and the Rashba spin orbit interaction inside the two dots are taken into account, Rashba spin-orbit interaction contribution are exhibited obviously in the double quantum dots system for the thermoelectric effect, by varying the temperature of the leads, we calculate the conductance (G) and the thermopower (S).

Keywords: quantum dots, Rashba spin-orbit interaction, conductance, thermopower
1. Introduction

The world today stands on the threshold of a huge scientific revolution, which is mainly nanotechnology. This technology represents a huge leap in all sciences, especially engineering and medicinal [1-3], and in the applications of various physical [4], chemical and biological sciences [5], etc. It is possible to synthesize nanoparticles smaller than the Bohr diameter of the electrons in them and thus subject to quantum wavelength [6], for example small groups of atoms called quantum dots which are known to be good systems for basic studies of transport phenomena using thermal effects [7] due to their small size and unique properties. The thermoelectric response [8] is very sensitive to small thermal biases [9], where thermal currents are controlled using Nano devices, it is vital to improve the operation of electronic devices. There are different methods for investigating the scattered heat in electronic devices or even taking advantage of it. It is possibility to convert it into energy by heat engines [10-12] most of the research has focused on the transmission properties [13] of the microscope systems [14]. The Anderson's model of two dots [15-16] has been used to combined the two dots with non-magnetic electrodes, normal poles and taking into account the thermal transfer caused by the temperature gradient across the interferometer [1]. At the theoretical level, interest in the transport properties of parallel quantum dot is not new[17] and has focused mainly on interference effects [18], which can be understood by using Coulomb Blockade and conduction via the double diaphragm system in which the current through the quantum dot is restricted to the tunneling barrier that can be controlled by gate voltages [19]. When the quantum interference of tunneling electrons results in a circular electric current that can make the magnetic states of the device polarized upward or downward [20]. In this paper, we study the thermal effects in a system consisting of two dots connected to normal metal leads in parallel and by applying an external magnetic field perpendicular to the system. The system was described by Anderson Hamiltonian model for two impurities, and we used the green function in a detailed derivation of the permeability factor in the form of a matrix to calculate the conductivity and thermopower. Recently, the spin-orbit-Rashpa interaction(RSOI) [21] has attracted great attention [22-23] because it plays an important role in the distinguished results is the emergence of the possibility of polarizing electrons flowing along the direction of the field [24], and that the direction and strength of the spin polarization in quantum dot can be well controlled. Or manipulation it through bias and Rashba interaction to generate a quantum spin current which [25] represents a great challenge, as well as the development of a quantum computer [26] as spinning the electron is an ideal candidate for qubits to achieve a quantum computer in the future [27].
The organization of the rest of the paper is as follows. In Sec. 2. we present the model and the discussion of some aspects of our execution. The results are discussed in Sec. 3. We end up with compendium and conclusions in Sec. 4.

2. Model and Hamiltonian

In this paper we present a schematic presentation that we will use in the study of thermal effects in a system consisting of two quantum dots linked into two poles in parallel and under the influence of a constant magnetic field, as illustrate in Fig. 1.

![Diagram](image)

Fig. 1: A quantum interferometer base on two quantum dots. Both dots are tunnel-coupled with each other, $t_c$, and the left and right leads. The tunneling amplitudes between the dots and leads are denoted by $\Gamma_{1\sigma}$ and $\Gamma_{2\sigma}$. The energy levels positions in each dot are $E_1$ and $E_2$ relative to the Fermi energy respectively.

The parallel connection of the two dots can provide two arms to transfer electrons from the left pole (L) to the right pole (R). In these two arms, the two quantum dots that each one has contained one energy level that included, and there is a link between the two dots (tc) in the form of a tunnel and affects. The system is under a magnetic field (B) and the rate of the transmission in the upper arms is ($\Gamma_1$) and ($\Gamma_2$) in the lower arm. Therefore, the Hamiltonian system can be described as follows [28]:

$$H = H_{leads} + H_{dots} + H_{interaction}$$

Where,

$$H = \sum_k \epsilon_k c_k^\dagger c_k + \sum_i \epsilon_i d_i^\dagger d_i + \sum_{k\alpha\sigma} t_{k\alpha\sigma} (c_{k\alpha\sigma}^d d_{i\sigma} + d_{i\sigma}^d c_{k\alpha\sigma}) - t_c \sum_\sigma (d_{1\sigma}^d d_{2\sigma} + d_{2\sigma}^d d_{1\sigma})$$

The first term of the equation (1) represents the Hamiltonian of the left and right poles. The second term is the Hamiltonian for the two quantum dots, while the last term represents the Hamiltonian between the quantum dot and the poles. Since $K$ is the wave vector of electrons inside.
the electrodes and \( \alpha \) indicates the left and right poles (L and R), and \( \sigma \) is the direction of spinning electrons (\( \sigma = \uparrow \downarrow \)). \( \epsilon_{K\alpha} \) represents the energy levels of the electrodes, \( C_{k\alpha}^+ \) and \( C_{k\alpha}^- \) are the (creation) annihilation catabolism and growth influences of conduction electrons respectively at the \( \alpha \) electrode, \( i \) represents the first and second numeral dot and \( \epsilon_i \) is the energy levels within the quantum dot, \( (d^+_i) \) and \( d_i \) are the catabolism and growth effect of the electrons at the quantum dot, respectively. Where \( a_s t_{\alpha i\sigma} \) is the interaction energy between the quantum dot and the electrodes (representing the element of the electron transport matrix between the quantum dot and the pole can be written in the form

\[
L_{11} = \frac{T}{\hbar} \sum \int dE \left( -\frac{df_{\alpha}(E)}{dE} \right)_T T_{\sigma}(\omega)
\]

The equilibrium Fermi function \( f_{\alpha}(E) \) of the lead \( \alpha \) in the above equations is represent in the mathematical formula below:

\[
f(E) = \left( 1 + e^{\frac{(E-\mu_\alpha)}{kT}} \right)^{-1}
\]

Where \( \mu_\alpha \) is the chemical potential, and \( T_\alpha \) is the temperature (Kelvin, K). The thermopower \( S \) of a system is represented as [28]:

\[
s = -\frac{\Delta V}{\Delta T} \bigg|_{i=0} = -\frac{1}{eT} \cdot \frac{L_{12}}{L_{11}}
\]

Where \( \Delta V \) is the bias voltage between two leads. The transmission function is given by [29]

\[
T(\omega) = Tr(G^A(\omega)I^R G^R(\omega)I^L)
\]

Where the retarded Green function of the quantum dot is given by:

\[
G_{ij}^R(t, t') = -i\theta(t) \left< d_{i\sigma}(t) d_{j\sigma}(t') \right>
\]

And \( G^A(\omega) = [G^R(\omega)]^* \)

\( \theta(t) \) is the units steep function, and \( d_{i\sigma}(t) \) is the annihilation operator of the quantum dot. \( d^+_i(\ell) \) is the creation operator of electron to be present on the dots. As we can use the relation for the operator \( d_{i\sigma}(t) \) which is given by:
\[ C_{k\alpha\sigma}(t) = e^{iht}C_{k\alpha\sigma}e^{-iht} \]  
(10)
\[ \frac{d}{dt} C_{k\alpha\sigma}(t) = iHe^{iht}C_{k\alpha\sigma}e^{-iht} + e^{iht}C_{k\alpha\sigma}(-iH)e^{-iht} \]  
(11)
Differentiate equation (9) and using equation (11) we get:
\[ i \frac{d}{dt} C_{k\alpha\sigma}(t) = \delta(t) \cdot \delta_{ij} + \theta(t) \cdot <\{ e^{iht}[C_{k\alpha\sigma}, H] e^{-iht}, d_{\sigma}^{+}(t) \}> \]  
(12)
To solve the kinematic equation for this system we will use the retarded Green function and it’s complex conjugate:
\[ G_{11} = \frac{\left(\omega - E_2 + i \frac{\Gamma}{2}\right)}{\frac{i}{4} \cos^2 \frac{\varphi}{2} + \left(\omega - E_1 + i \frac{\Gamma}{2}\right) \left(\omega - E_2 + i \frac{\Gamma}{2}\right)} \]
\[ G_{21} = \frac{-i \frac{\Gamma}{2} \cos \frac{\varphi}{2}}{\frac{i}{4} \cos^2 \frac{\varphi}{2} + \left(\omega - E_1 + i \frac{\Gamma}{2}\right) \left(\omega - E_2 + i \frac{\Gamma}{2}\right)} \]
\[ G_{22} = \frac{\left(\omega - E_1 + i \frac{\Gamma}{2}\right)}{\frac{i}{4} \cos^2 \frac{\varphi}{2} + \left(\omega - E_1 + i \frac{\Gamma}{2}\right) \left(\omega - E_2 + i \frac{\Gamma}{2}\right)} \]
\[ G_{21} = \frac{-i \frac{\Gamma}{2} \cos \frac{\varphi}{2}}{\frac{i}{4} \cos^2 \frac{\varphi}{2} + \left(\omega - E_1 + i \frac{\Gamma}{2}\right) \left(\omega - E_2 + i \frac{\Gamma}{2}\right)} \]  
(13)
By re-writing the retreaded green function matrix elements as:
\[ G^R = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \]
We got:
\[ G^R = \frac{1}{\frac{i}{4} \cos^2 \frac{\varphi}{2} + \left(\omega - E_1 + i \frac{\Gamma}{2}\right) \left(\omega - E_2 + i \frac{\Gamma}{2}\right)} \begin{bmatrix} \left(\omega - E_2 + i \frac{\Gamma}{2}\right) & -i \frac{\Gamma}{2} \cos \frac{\varphi}{2} \\ -i \frac{\Gamma}{2} \cos \frac{\varphi}{2} & \left(\omega - E_1 + i \frac{\Gamma}{2}\right) \end{bmatrix} \]  
(14)
The tunnel coupling matrices for symmetric leads is given by the equations:
\[ r^L = \frac{r}{2} \begin{bmatrix} 1 & e^{i\frac{\varphi}{2}} \\ e^{-i\frac{\varphi}{2}} & 1 \end{bmatrix}, \quad r^R = \frac{r}{2} \begin{bmatrix} 1 & e^{-i\frac{\varphi}{2}} \\ e^{i\frac{\varphi}{2}} & 1 \end{bmatrix} \]  
(15)
Where \( \varphi = 2\pi \Phi/\Phi_0 \) is the phase displacement resulting from the magnetic flux \( \Phi \) at the two dots and \( \Phi_0 \) represents the quantum flux and equals \( h/e \) where \( h \) is Planck’s constant and \( e \) is the charge of the electron. By inserting the spin effect an Rashba orbital effect, which can be defined as \( \varphi_{Rt} = \)
\( \beta L_i m^*/\hbar^2 \), since \( \beta \) is the RSOI strength, and \( L_i \) is the length of dot \( i \), \( m^* \) represents the effective mass of the electron and therefore the phase \( \Phi_\sigma \) is equal to \( (\varphi - \sigma \varphi_{Ri}) \) if \( \sigma = \pm 1 \) for both the upper and lower twine, respectively. In order to find the permeability factor \( T(\omega) \), which is defined in equation (8). The first diagonal term in the result of the multiplication \( G^A(\omega) G^R(\omega) \Gamma^L \) is:

\[
T_{11} = (\omega - E_2)^2 + \frac{\Gamma^2}{4} - i\Gamma \cos \frac{\varphi}{2} e^{-i\frac{\varphi}{2}} (\omega - E_2 - i \frac{\Gamma}{2}) + i \frac{\Gamma}{2} \cos \frac{\varphi}{2} e^{i\frac{\varphi}{2}} (\omega - E_2 + i \frac{\Gamma}{2})
+ \frac{\Gamma^2}{2} \cos^2 \frac{\varphi}{2} + (\omega - E_2 - i \frac{\Gamma}{2}) (\omega - E_1 + i \frac{\Gamma}{2}) e^{-i\varphi} + i \frac{\Gamma}{2} \cos \frac{\varphi}{2} (\omega - E_1 + i \frac{\Gamma}{2}) e^{-\frac{\varphi}{2}}
\]

And the last element form is:

\[
T_{22} = (\omega - E_1)^2 + \frac{\Gamma^2}{4} + \frac{\Gamma^2}{2} \cos^2 \frac{\varphi}{2} + i \frac{\Gamma}{2} \cos \frac{\varphi}{2} (\omega - E_1 + i \frac{\Gamma}{2}) e^{i\frac{\varphi}{2}}
+ (\omega - E_2 - i \frac{\Gamma}{2}) (\omega - E_1 + i \frac{\Gamma}{2}) e^{i\varphi} - i\Gamma \cos \frac{\varphi}{2} e^{i\frac{\varphi}{2}} (\omega - E_2 - i \frac{\Gamma}{2}) + i \frac{\Gamma}{2} \cos \frac{\varphi}{2} e^{-i\frac{\varphi}{2}} (\omega - E_1 + i \frac{\Gamma}{2}) e^{-\frac{\varphi}{2}}
\]

So, the transmission coefficients, \( T(\omega) \) is:

\[
T(\omega) = T_{11} + T_{22} = \frac{\Gamma^2 \left( \cos^2 \frac{\varphi}{2} (\omega - \bar{E})^2 + \left( \frac{\Delta E}{2} \right)^2 \sin^2 \frac{\varphi}{2} \right)}{\left[ (\omega - \bar{E})^2 - \left( \frac{\Delta E}{2} \right)^2 - \frac{\Gamma^2}{4} \sin^2 \frac{\varphi}{2} \right]^2 + [\Gamma(\omega - \bar{E})]^2}
\]

Where \( \Delta E = E_2 - E_1 \), \( \bar{E} = \frac{E_1 + E_2}{2} \)

3. Result and discussion

3.1 Calculation of transmission coefficients, \( T(\omega) \)

The symmetric case is considered in the following calculations, we set \( E_1 = E_2 = E_0 \), and the line width matrix elements between the dots and leads as \( \Gamma_{11}^\alpha = \Gamma_{22}^\alpha = \Gamma (\alpha = L, R) \) also we used atomic units, \( e = \hbar = K_B = 1 \). As can be seen in Fig. 2, the relation between \( T(\omega) \) and the Rashba spin orbit interaction which induced a phase \( \Phi_R \), the peak is shifted towards the greater \( \Phi_R \) for \( \Phi = 0.9\pi \) then the peak of \( \Phi = \pi \). Fig. 3 shows the transmission coefficient as a function of the phase \( \Phi_R \), and we made the image of the lower spin (\( \sigma = \downarrow \)) inverted in order to distinguish between the two spin.
Fig. 2: The transmission spectral of the electron as a function of $\Phi_R$ between the quantum dot and the right Lead at $\Gamma = 1, \omega = 0.1, E_1 = E_2 = 0, \Phi = 0.9\pi$.

Fig. 3: It represents the relationship between the electron transmission coefficient as a function of the interaction of the RSO with the values $\Gamma = 1, \omega = 0.1, E_1 = E_2 = 0$. 
3.1 Calculation of thermal conductance, $G(\omega)$

In this sub-section we discuss the thermal conductance as a function of tunneling energy, $t_c$, for different parameters as shown in Fig. 4. The figure represent the effect of different $\Gamma$, and the results show that more increasing peak and more broadening as that in reference[23]. In Fig. 5 we show the conductance with different spin at Rashba interaction $\Phi_R = \pi/4$, magnetic flux phase $\Phi = \pi/2$, temperature $T = 1$ and $\Gamma = 1$ also $E_1 = E_2 = 0$, the blue line represents the spin – up and the red line is spin – down, also we draw the total spin as in black line. Fig. 6 shows the dependent of conductance on the coupling energy with different magnetic flux phase at fixed RSOI, $\Phi_R = \pi/4$, $T = 1$ and line width $\Gamma = 1$.

![Fig. 4: The thermal conductance as a function of the coupling energy $t_c$ for three different values of line width $\Gamma(1,2,3)$](image-url)
3.3 Calculations of thermopower (S)

The dependence of thermopower on temperature, T, is calculating and is shown in Fig. 7, where we use the parameters $\Gamma = 1.5$, $t_c = 0$, $E_1 = E_2 = 0$ and the RSOI is zero. We see from the figure there is a small change in the maximum value when we change the magnetic flux which induced
a phase $\Phi = \pi, \pi/2$ and $\pi/4$ the peak is at the temperature, $T = 10^\circ K$. Fig. 8 shows the thermopower as a function of temperature with difference $\Gamma$, it represents that for small $\Gamma$, the peak is shifted to the less temperature and less broadening and vice versa. Also we study the effect of spin and it is shown in Fig. 9 for values $\Phi_R = \pi/4$, and $\Phi = \pi/2$. The blue line represents the spin – up and the red line spin – down, while the other parameter are $\Gamma = 1$, $t_c = 0$ and $E_1 = E_2 = 0$. The evolution of thermopower of a function of interdot coupling, $t_c$, is sketch in Fig. 10 for different spin. The spin – up thermopower has a peak position at $t_c = 3$ a.u, where spin – down power peak at $t_c = 1.8$ a.u, where the other parameter used in this calculation are $\Phi_R = 5\pi/4, T = 1, \Gamma = 1, \Phi = \pi/2$ and $E_1 = E_2 = 0$. The effect of quantum flux which introduction the phase $\Phi = \pi, \pi/2$ and $\pi/4$ are shown in Fig. 11, while Fig. 12 shows the effect of changing the parameter $\Gamma$ on the thermopower, we see all the curve are intersect at $t_c = 2.5$ a.u, which gives zero thermopower.

Fig. 7: Relationship between thermopower ($S$) as a function of temperature at values $\Gamma = 1.5$, $t_c = 0, E_1 = E_2 = 0, \Phi_R = 0$. 
Fig. 8: Relationship between thermopower (S) as a function of temperature at values $\Phi = 0$, $t_c = 0, E_1 = E_2 = 0, \Phi_R = 0$.

Fig. 9: Relationship between thermopower (S) dependent on RSO $\Phi_R = \pi/4$, Coefficients of magnetic field flux $\Phi = \frac{\pi}{2}$, as a function of temperature linewidth $\Gamma = 1$, for energy levels $E_1 = E_2 = 0$, and coupling energy $t_c = 0$. 

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Fig. 10: Relationship between thermopower ($S$) dependent on RSO $\Phi_R = 5\pi/4$ as function of the interdot coupling strength $t_c$, and linewidth $\Gamma = 1$, for energy levels $E_1 = E_2 = 0$.

Fig. 11: Relationship between thermopower ($S$) as function of the interdot coupling strength $t_c$, and Transmission rate (linewidth) $\Gamma = 1$, for energy levels $E_1 = E_2 = 0$. 

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4. Conclusions

Electronic transport has been studied through a sub-interferometer of double quantum dots with one energy level connected to two identical leads $\Gamma_{11}^{\alpha} = \Gamma_{22}^{\alpha} = \Gamma$ ($\alpha = L, R$). We take into account both the magnetic flux $\Phi$ applied to the circuit and the Rashba spin-orbit interaction inside the two dots. Our offer the contribution of the RSOI in quantum dots system dual on the thermoelectric effect, by changing the temperature of leads and spin electron and the rate of the transition and their impact on the conductance (G) and thermopower (S).
References


المستخلص

النموذج النظري لانتقال الإلكترون في مطياف ذو فرعين قد تم إعداده في هذا البحث وقد تم تضمين نقطة كمية في كل فرع من فروع المطياف وقد تم تطبيق مجال مغناطيسي خارجي ذو فيض $\Phi$ على الدائرة بوجود تفاعلات البرم والمدار لراشبا في النقطتين الكميتين. ان تفاعل البرم–المدار لراشبا يظهر واضحا في النقطتين الكميتين نتيجة لتأثير الحراري من خلال تغير درجة حرارة الأقطاب وقد تم حساب التوصيلية، $G$, مع القدرة الحرارية، $S$. 

انسجور الالكترون خلال مطياف ذو فرعين بوجود تفاعل البرم–المدار لراشبا

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