

Decomposition matrices for the spin characters of S_{26} Modulo 11

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Abstract

We found in this paper the 11-decomposition matrices for the spin characters of S_{26} , which are a relationship between modular and ordinary characters

1. Introduction

For any group there are three kinds of characters ordinary, modular (for a given prime p), and projective (for S_n called spin). The decomposition matrix is the relation between the ordinary and modular characters for a given prime p (see[1], [2]). Spin characters of S_n can be written as a liner combination with non-negative coefficients of the irreducible spin characters [2]. The case in $n = 25$ found by M. M. Jawad [3]. In this paper we use the techniques as given in [4]. The matrix for this spin from degree (247,207)[2], [5]. There are 50 blocks, the blocks B_1, B_2 are of defect two, the blocks $B_3, \dots, B_{17}$ are of defect one the others of defect zero. The notations used in this section can be found in [6].

Lemma (1). Brauer trees of blocks $B_3, B_4, \dots, B_{17}$ [7] are:

$$\begin{aligned}
 \langle 24,2 \rangle^* _ \langle 13,11,2 \rangle &= \langle 13,11,2 \rangle' _ \langle 13,10,2,1 \rangle^* _ \langle 13,8,3,2 \rangle^* _ \langle 13,7,4,2 \rangle^* _ \langle 13,6,5,2 \rangle^*, \\
 \langle 23,2,1 \rangle _ \langle 13,12,1 \rangle &\searrow \langle 12,11,2,1 \rangle^* \swarrow \langle 12,8,3,2,1 \rangle _ \langle 12,7,4,2,1 \rangle _ \langle 12,6,5,2,1 \rangle \\
 \langle 23,2,1 \rangle' _ \langle 13,12,1 \rangle' &\swarrow \langle 12,11,2,1 \rangle^* \searrow \langle 12,8,3,2,1 \rangle' _ \langle 12,7,4,2,1 \rangle' _ \langle 12,6,5,2,1 \rangle', \\
 \langle 25,5 \rangle^* _ \langle 16,10 \rangle^* _ \langle 11,10,5 \rangle &= \langle 11,10,5 \rangle' _ \langle 10,9,5,2 \rangle^* _ \langle 10,8,5,2 \rangle^* _ \langle 10,,7,5,4 \rangle^*, \\
 \langle 21,3,2 \rangle _ \langle 14,10,2 \rangle _ \langle 13,10,3 \rangle &\searrow \langle 11,10,3,2 \rangle^* \swarrow \langle 10,7,4,3,2 \rangle _ \langle 10,6,5,3,2 \rangle \\
 \langle 21,3,2 \rangle' _ \langle 14,10,2 \rangle' _ \langle 13,10,3 \rangle' &\swarrow \langle 11,10,3,2 \rangle^* \searrow \langle 10,7,4,3,2 \rangle' _ \langle 10,6,5,3,2 \rangle', \\
 \langle 20,6 \rangle^* _ \langle 17,9 \rangle^* _ \langle 11,9,6 \rangle &= \langle 11,9,6 \rangle' _ \langle 10,9,6,1 \rangle^* _ \langle 9,8,6,3 \rangle^* _ \langle 9,7,6,4 \rangle^*, \\
 \langle 20,5,1 \rangle _ \langle 16,9,1 \rangle _ \langle 12,9,5 \rangle &\searrow \langle 11,9,5,1 \rangle^* \swarrow \langle 9,8,5,3,1 \rangle _ \langle 9,7,5,4,1 \rangle \\
 \langle 20,5,1 \rangle' _ \langle 16,9,1 \rangle' _ \langle 12,9,5 \rangle' &\swarrow \langle 11,9,5,1 \rangle^* \searrow \langle 9,8,5,3,1 \rangle' _ \langle 9,7,5,4,1 \rangle',
 \end{aligned}$$

$$\begin{aligned}
& \langle 19,7 \rangle^* _ \langle 18,8 \rangle^* _ \langle 11,8,7 \rangle = \langle 11,8,7 \rangle' _ \langle 10,8,7,1 \rangle^* _ \langle 9,8,7,2 \rangle^* _ \langle 8,7,6,5 \rangle^*, \\
& \langle 19,6,1 \rangle _ \langle 17,8,1 \rangle _ \langle 12,8,6 \rangle \quad \backslash \quad \langle 11,8,6,1 \rangle^* _ \langle 9,8,6,2,1 \rangle _ \langle 8,7,6,4,1 \rangle \\
& \langle 19,6,1 \rangle' _ \langle 17,8,1 \rangle' _ \langle 12,8,6 \rangle' \quad / \quad \langle 11,8,6,1 \rangle^* _ \langle 9,8,6,2,1 \rangle' _ \langle 8,7,6,4,1 \rangle', \\
& \langle 19,5,2 \rangle _ \langle 16,8,2 \rangle _ \langle 13,8,5 \rangle \quad \backslash \quad \langle 11,8,5,2 \rangle^* _ \langle 10,8,5,2,1 \rangle _ \langle 8,7,5,4,2 \rangle \\
& \langle 19,5,2 \rangle' _ \langle 16,8,2 \rangle' _ \langle 13,8,5 \rangle' \quad / \quad \langle 11,8,5,2 \rangle^* _ \langle 10,8,5,2,1 \rangle' _ \langle 8,7,5,4,2 \rangle', \\
& \langle 19,4,2,1 \rangle^* _ \langle 15,8,2,1 \rangle^* _ \langle 13,8,4,1 \rangle^* _ \langle 12,8,4,2 \rangle^* _ \langle 11,8,4,2,1 \rangle = \\
& \langle 11,8,4,2,1 \rangle' _ \langle 8,6,5,4,2 \rangle^*, \\
& \langle 18,6,2 \rangle _ \langle 17,7,2 \rangle _ \langle 13,7,6 \rangle \quad \backslash \quad \langle 11,7,6,2 \rangle^* _ \langle 10,7,6,2,1 \rangle _ \langle 8,7,6,3,2 \rangle \\
& \langle 18,6,2 \rangle' _ \langle 17,7,2 \rangle' _ \langle 13,7,6 \rangle' \quad / \quad \langle 11,7,6,2 \rangle^* _ \langle 10,7,6,2,1 \rangle' _ \langle 8,7,6,3,2 \rangle', \\
& \langle 18,5,3 \rangle _ \langle 16,7,3 \rangle _ \langle 14,7,5 \rangle \quad \backslash \quad \langle 11,7,5,3 \rangle^* _ \langle 10,7,5,3,1 \rangle _ \langle 9,7,5,3,2 \rangle \\
& \langle 18,5,3 \rangle' _ \langle 16,7,3 \rangle' _ \langle 14,7,5 \rangle' \quad / \quad \langle 11,7,5,3 \rangle^* _ \langle 10,7,5,3,1 \rangle' _ \langle 9,7,5,3,2 \rangle', \\
& \langle 18,5,2,1 \rangle^* _ \langle 16,7,2,1 \rangle^* _ \langle 13,7,5,1 \rangle^* _ \langle 12,7,5,2 \rangle^* _ \langle 11,7,5,2,1 \rangle = \\
& \langle 11,7,5,2,1 \rangle' _ \langle 8,7,5,3,2,1 \rangle^*, \\
& \langle 17,4,3,2 \rangle^* _ \langle 15,6,3,2 \rangle^* _ \langle 14,6,4,2 \rangle^* _ \langle 13,6,4,3 \rangle^* _ \langle 11,6,4,3,2 \rangle = \\
& \langle 11,6,4,3,2 \rangle' _ \langle 10,6,4,3,2,1 \rangle^*, \\
& \langle 16,4,3,2,1 \rangle _ \langle 15,5,3,2,1 \rangle _ \langle 14,5,4,2,1 \rangle _ \langle 13,5,4,3,1 \rangle _ \langle 12,5,4,3,2 \rangle \quad \backslash \quad \langle 11,5,4,3,2,1 \rangle^* \\
& \langle 16,4,3,2,1 \rangle' _ \langle 15,5,3,2,1 \rangle' _ \langle 14,5,4,2,1 \rangle' _ \langle 13,5,4,3,1 \rangle' _ \langle 12,5,4,3,2 \rangle' \quad / \quad \langle 11,5,4,3,2,1 \rangle^* \\
& \text{respectively.}
\end{aligned}$$

Proof. For B_4 , we use the $(r, \bar{r})$ -inducing p.i.s. of $D_{25}, D_{41}, D_{43}, D_{44}, \dots, D_{48}$, of S_{25} to S_{26} we get $k_1, k_2, d_{70}, d_{71}, \dots, d_{75}$, and since it's associate (see [2]), so $\langle 23,2,1 \rangle \neq \langle 23,2,1 \rangle'$ it follows that k_1 splits or there are two columns:

$$Y_1 = a_1 \langle 23,2,1 \rangle + a_2 \langle 13,12,1 \rangle + a_3 \langle 12,11,2,1 \rangle^* + a_4 \langle 12,8,3,2,1 \rangle + a_5 \langle 12,7,4,2,1 \rangle + a_6 \langle 12,6,5,2,1 \rangle,$$

$$Y_2 = a_1 \langle 23,2,1 \rangle' + a_2 \langle 13,12,1 \rangle' + a_3 \langle 12,11,2,1 \rangle^* + a_4 \langle 12,8,3,2,1 \rangle' + a_5 \langle 12,7,4,2,1 \rangle' + a_6 \langle 12,6,5,2,1 \rangle',$$

so that $a_1, a_2, \dots, a_6 \in \{0,1\}[7]$. Let $a_1 = 1$. Since $\langle 23,2,1 \rangle \downarrow S_{25} \cap \langle 12,8,3,2,1 \rangle \downarrow S_{25}$ has no i.m.s, so $a_4 = 0$. The same way, we find $a_5, a_6 = 0$. But $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $a_1, a_2 = 1, a_3 = 0$. It follows that $k_1 = d_{66} + d_{67}$. Since B_4 of defect one. So $k_2 = d_{68} + d_{69}$ [3]. In the same way B_6 , we use $(r, \bar{r})$ -inducing and to get $k_1, k_2, \dots, k_5$. Since it's associate [5]. So k_1 divided or there are columns:

$$Y_1 = a_1 \langle 21,3,1 \rangle + a_2 \langle 14,10,2 \rangle + a_3 \langle 13,10,3 \rangle + a_4 \langle 11,10,3,2 \rangle^* + a_5 \langle 10,7,4,3,2 \rangle + a_6 \langle 10,6,5,3,2 \rangle,$$

$$Y_2 = a_1 \langle 21,3,1 \rangle' + a_2 \langle 14,10,2 \rangle' + a_3 \langle 13,10,3 \rangle' + a_4 \langle 11,10,3,2 \rangle^* + a_5 \langle 10,7,4,3,2 \rangle' + a_6 \langle 10,6,5,3,2 \rangle', \quad a_1, a_2, \dots, a_6 \in \{0,1\}. \text{ Let } a_1 = 1, \text{ and } \langle 21,3,2 \rangle \downarrow S_{25} \cap \langle 13,10,3,2 \rangle \downarrow S_{25} \text{ has}$$

no i.m.s. So $a_3 = 0$. In the same way we get $a_4, a_5, a_6 = 0$. Then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $a_1, a_2 = 1, a_3 = 0$ so $k_1 = d_{81} + d_{82}$. In the same we divided k_2, k_5 to $d_{83}, d_{84}, d_{89}, d_{90}$ respectively. But B_6 of defect one so k_3, k_4 splits to $d_{85}, d_{86}, d_{87}, d_{88}$. In the way we prove the blocks B_{11}, B_{13}, B_{14} . For B_8 , we use the $(r, \bar{r})$ -inducing to get on $d_{96}, d_{97}, k_1, k_2, d_{102}, \dots, d_{105}$. Since it's associate so k_1 divided or there Y_1, Y_2 such that:
 $Y_1 = a_1 \langle 16, 9, 1 \rangle + a_2 \langle 12, 9, 5 \rangle + a_3 \langle 11, 9, 5, 1 \rangle^* + a_4 \langle 9, 8, 5, 3, 1 \rangle + a_5 \langle 9, 7, 5, 4, 1 \rangle,$
 $Y_2 = a_1 \langle 16, 9, 1 \rangle' + a_2 \langle 12, 9, 5 \rangle' + a_3 \langle 11, 9, 5, 1 \rangle^* + a_4 \langle 9, 8, 5, 3, 1 \rangle' + a_5 \langle 9, 7, 5, 4, 1 \rangle',$
 $a_1, a_2, \dots, a_5 \in \{0, 1\}$. In the same above $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $a_1, a_2 = 1$ so $k_1 = d_{96} + d_{97}$. But it's of defect one so $k_2 = d_{100} + d_{101}$. In the same way mentioned above we discuss the block B_{10} and B_{17} . The other we found it by use the $(r, \bar{r})$ -inducing from S_{25} to S_{26} directly. Finally we find the Brauer trees for the blocks $B_3, B_4, \dots, B_{17}$ of the decomposition that we found [7].

Lemma (2). Decomposition matrix for the block B_2 of a double [7] is a table (1).

$\langle 25,1 \rangle^*$	1																			
$\langle 23,3 \rangle^*$	1	1																		
$\langle 22,3,1 \rangle$		1	1																	
$\langle 20,3,2,1 \rangle^*$			1	1																
$\langle 18,4,3,1 \rangle^*$				1	1															
$\langle 17,5,3,1 \rangle^*$					1	1														
$\langle 16,6,3,1 \rangle^*$					1	1	1													
$\langle 15,7,3,1 \rangle^*$				1	1		1	1												
$\langle 14,12 \rangle^*$		1							1											
$\langle 14,11,1 \rangle$	1	1	1						1	1										
$\langle 14,9,2,1 \rangle^*$			1	1						1	1									
$\langle 14,8,3,1 \rangle^*$				1				1			1	1								
$\langle 14,7,4,1 \rangle^*$							1	1				1	1							
$\langle 14,6,5,1 \rangle^*$							1						1							
$\langle 13,9,3,1 \rangle^*$									1	1	1		1							
$\langle 12,11,3 \rangle$	1								1	1					1					
$\langle 12,10,3,1 \rangle^*$									2	1				1	2	2				
$\langle 12,9,3,2 \rangle^*$												1		1	1	2	1			
$\langle 12,7,4,3 \rangle^*$												1	1				1	1		
$\langle 12,6,5,3 \rangle^*$													1					1		
$\langle 11,9,3,2,1 \rangle$															1	1	1		1	
$\langle 11,7,4,3,1 \rangle$																1	1	1	1	
$\langle 11,6,5,3,1 \rangle$																		1	1	
$\langle 9,7,4,3,2,1 \rangle^*$																1			1	
$\langle 9,6,5,3,2,1 \rangle^*$																			1	
$\langle 7,6,5,4,3,1 \rangle^*$																			1	
	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}	d_{57}	d_{58}	d_{59}	d_{60}

table (1)

Proof. Using $(r, \bar{r})$ -inducing of p.i.s. $D_{21}, D_{23}, D_3, D_4, \dots, D_8, D_{39}, D_{10}, D_{11}, D_{45}, D_{47}, D_{158}, D_{39}, D_{16}, D_{53}, D_{55}, D_{19}, D_{20}$ of S_{25} to S_{26} , we got $d_{41}, d_{42}, \dots, d_{60}$ respectively. Since $(d_{45} - d_{46}) \downarrow_{(1,0)} S_{25}$ is not p.s., so $d_{46} \notin d_{45}$, consequently we get the table (1).

Theorem (3). Decomposition matrix for spin character of S_{26} the block B_1 is table (2).

[illegible]

table (2)

Proof. By using $(r, \bar{r})$ -inducing of S_{25} to S_{26} , we get approximation matrix in table (3).

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table(3)

Since $(k_4 - k_5) \downarrow_{(8,4)} S_{25}$ is not p.s., so $k_5 \not\subset k_4$. However $k_9 \subset c$ we prove that by the way of contradiction. Let $(\langle 14,8,4 \rangle - \langle 15,6,5 \rangle)$ is m.s. for S_{26} but $\langle 14,8,4 \rangle - \langle 15,6,5 \rangle \downarrow_{(8,4)} S_{25}$ is not m.s., so $k_6 = c - k_9$.

From [3] k_1 must split to d_1, d_2 . Also $\langle 20,4,2 \rangle \neq \langle 20,4,2 \rangle'$ on $(11, \alpha)$ -regular classes (see[4]). So $k_2 = d_5 + d_6$ or there are two columns. If there are two columns Y_1, Y_2 then we have the approximation matrix as above, to describe it's such that $\langle 21,4,1 \rangle \downarrow S_{25}$ has 4 of i.m.s. and, from the table(3) we get $a_1 \in \{0,1,2\}$, in the same way we get $a_3, a_4, a_{11}, a_{18}, a_{24} \in \{0,1\}, a_2, a_5, a_6, a_{21}, a_{22}, a_{23} \in \{0,1,2\}, a_7, a_8, a_{15}, a_{17}, a_{19} \in \{0,1, \dots, 4\}, a_{10}, a_{13} \in \{0,1, \dots, 5\}, a_9, a_{12}, a_{14}, a_{16}, a_{20} \in \{0,1, \dots, 6\}$. Take $a_2 \in \{1,2\}$ (if $a_2 = 0$ then we have contradiction) and, since $\langle 20,4,2 \rangle \downarrow S_{25} \cap \langle 17,5,4 \rangle \downarrow S_{25}$ has no i.m.s, so we have $a_4 = 0$ by counting the intersections we get on $a_5, a_6, a_{11}, a_{13}, a_{14}, \dots, a_{24}$ are equal to zero, and since inducing m.s. is m.s. [8] we have:

$$(\langle 21,4 \rangle - \langle 15,10 \rangle + \langle 11,10,4 \rangle^*) \uparrow^{(1,11)} S_{26} \Rightarrow a_1 \geq a_7 \quad (1)$$

$$(\langle 15,10 \rangle - \langle 21,4 \rangle) \uparrow^{(1,11)} S_{26} \Rightarrow a_7 \geq a_1. \text{ Hence } a_1 = a_7 \quad (2)$$

$$(\langle 20,4,1 \rangle^* - \langle 15,9,1 \rangle + \langle 12,9,4 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_2 \geq a_8 \quad (3)$$

$$(\langle 15,9,1 \rangle^* - \langle 20,4,1 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_8 \geq a_2. \text{ Hence } a_2 = a_8 \quad (4)$$

$$(\langle 14,8,3 \rangle^* - \langle 15,7,3 \rangle^* + \langle 16,6,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{12} \geq a_{10} \quad (5)$$

$$(\langle 15,7,3 \rangle^* - \langle 14,8,3 \rangle^* + \langle 13,9,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} \geq a_{12}. \text{ Hence } a_{10} = a_{12} \quad (6)$$

$$(\langle 20,3,2 \rangle^* - \langle 21,3,1 \rangle^* - \langle 18,4,3 \rangle^* + \langle 22,3 \rangle + \langle 17,5,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_2 \geq a_1 + a_3 \quad (7)$$

$$(\langle 18,4,3 \rangle + \langle 21,3,1 \rangle^* - \langle 20,3,2 \rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_3 + a_1 \geq a_2. \text{ Hence } a_2 = a_1 + a_3 \quad (8)$$

then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $Y_1 + Y_2 = n_1 k_2 + n_2 k_3 + n_3 k_9$, $n_2 = 0, n_1 \in \{1,2\}$, $n_3 \in \{0,1, \dots, 5\}$ or $n_2 = 1, n_1 \in \{0,1\}, n_3 \in \{0,1, \dots, 5\}$. So $k_2 = d_5 + d_6$.

Also $\langle 19,4,3 \rangle \neq \langle 19,4,3 \rangle'$ on $(11, \alpha)$ -regular classes. It follows that k_3 or k_4 splits or there are Y_1, Y_2 . Let $a_3 = 1$, and use the same way above, we get:

$$Y_1 = a_2 \langle 20,4,2 \rangle + \langle 19,4,3 \rangle + a_8 \langle 15,9,2 \rangle + a_9 \langle 15,8,3 \rangle + a_{10} \langle 15,7,4 \rangle + a_{12} \langle 14,8,4 \rangle,$$

$$Y_2 = a_2 \langle 20,4,2 \rangle' + \langle 19,4,3 \rangle' + a_8 \langle 15,9,2 \rangle' + a_9 \langle 15,8,3 \rangle' + a_{10} \langle 15,7,4 \rangle' + a_{12} \langle 14,8,4 \rangle'$$

Since

$$(\langle 20,4,1 \rangle^* - \langle 15,9,1 \rangle^* + \langle 12,9,4 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_2 \geq a_8 \quad (9)$$

$$(\langle 15,9,1 \rangle^* - \langle 20,4,1 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_8 \geq a_2. \text{ Hence } a_2 = a_8 \quad (10)$$

$$(\langle 15,7,3 \rangle^* - \langle 14,8,3 \rangle^* + \langle 13,9,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} \geq a_{12} \quad (11)$$

$$(\langle 14,8,3 \rangle^* - \langle 15,7,3 \rangle^* + \langle 16,6,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{12} \geq a_{10}. \text{ Hence } a_{10} = a_{12} \quad (12)$$

$$(\langle 15,8,2 \rangle - \langle 19,4,2 \rangle - \langle 13,8,4 \rangle + \langle 11,8,4,2 \rangle) \uparrow^{(9,3)} S_{26} \Rightarrow a_9 \geq a_3 + a_{10} \quad (13)$$

$$(\langle 19,4,2 \rangle + \langle 13,8,4 \rangle - \langle 15,8,2 \rangle) \uparrow^{(9,3)} S_{26} \Rightarrow a_3 + a_{10} \geq a_9.$$

$$\text{Hence } a_9 = a_3 + a_{10} \quad (14)$$

$$(\langle 14,8,3 \rangle^* - \langle 14,9,2 \rangle^* + \langle 14,10,1 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} + a_9 \geq a_2 \quad (15)$$

$$(\langle 14,9,2 \rangle^* - \langle 14,8,3 \rangle^* + \langle 13,9,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_2 \geq a_9 + a_{10}.$$

$$\text{Hence } a_2 = a_9 + a_{10}, \quad (16)$$

then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $a_2, a_3, a_8, a_9 = 1, a_{10}, a_{12} = 0$. So $k_3 = d_7 + d_8$.

Also $\langle 17,5,4 \rangle \neq \langle 17,5,4 \rangle'$ on $(11, \alpha)$ -regular classes. Suppose that $a_4 = 1$. Using the same technic then we get $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $a_4, a_5 = 1$. So k_4, k_5 are split to $d_9, d_{10}, d_{11}, d_{12}$ respectively.

For k_6 , we have $\langle 16,6,4 \rangle \neq \langle 16,6,4 \rangle'$. So it's splits or there are other Y_1, Y_2 . So let $a_5 \in \{1,2\}$. By restricting and inducing, we get $Y_1 = a_5 \langle 16,6,4 \rangle + a_9 \langle 15,8,3 \rangle + a_{10} \langle 15,7,4 \rangle + a_{11} \langle 15,6,5 \rangle + a_{12} \langle 14,8,4 \rangle$, $Y_2 = a_5 \langle 16,6,4 \rangle' + a_9 \langle 15,8,3 \rangle' + a_{10} \langle 15,7,4 \rangle' + a_{11} \langle 15,6,5 \rangle' + a_{12} \langle 14,8,4 \rangle'$ and since

$$(\langle 13,8,4 \rangle^* + \langle 19,4,2 \rangle^* - \langle 15,8,2 \rangle^*) \uparrow^{(9,3)} S_{26} \Rightarrow a_{12} \geq a_9 \quad (17)$$

$$(\langle 15,8,2 \rangle^* - \langle 13,8,4 \rangle^* + \langle 11,8,4,2 \rangle^*) \uparrow^{(9,3)} S_{26} \Rightarrow a_9 \geq a_{12}. \text{ Hence } a_9 = a_{12}, \quad (18)$$

then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $Y_1 + Y_2 = k_6 + nk_9, n \in \{0,1, \dots, 4\}$. So $k_6 = d_{13} + d_{14}$.

For k_8 , we have $\langle 15,9,2 \rangle \neq \langle 15,9,2 \rangle'$. Let $a_8 \in \{1, \dots, 4\}$. By restricting and inducing, we get: $Y_1 = a_8 \langle 15,9,2 \rangle + a_9 \langle 15,8,3 \rangle + a_{10} \langle 15,7,4 \rangle + a_{12} \langle 14,8,4 \rangle + a_{13} \langle 13,9,4 \rangle + a_{14} \langle 12,10,4 \rangle$, $Y_2 = a_8 \langle 15,9,2 \rangle' + a_9 \langle 15,8,3 \rangle' + a_{10} \langle 15,7,4 \rangle' + a_{12} \langle 14,8,4 \rangle' + a_{13} \langle 13,9,4 \rangle' + a_{14} \langle 12,10,4 \rangle'$ and, since

$$(\langle 15,10 \rangle + \langle 15,10 \rangle' + \langle 10,9,4,2 \rangle + \langle 10,9,4,2 \rangle' - \langle 11,10,4 \rangle^*) \uparrow^{(1,11)} S_{26} \Rightarrow 0 \geq a_{14}. \text{ Hence } a_{14} = 0 \quad (19)$$

$$(\langle 15,9,1 \rangle^* - \langle 12,9,4 \rangle^* + \langle 11,9,4,1 \rangle) \uparrow^{(10,2)} S_{26} \Rightarrow a_8 \geq a_{13} \quad (20)$$

$$(\langle 12,9,4 \rangle^* - \langle 15,9,1 \rangle^* + \langle 20,4,1 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{13} \geq a_8. \text{ Hence } a_8 = a_{13} \quad (21)$$

$$(\langle 15,8,2 \rangle^* - \langle 13,8,4 \rangle^* + \langle 11,8,4,2 \rangle) \uparrow^{(9,3)} S_{26} \Rightarrow a_9 \geq a_{12} \quad (22)$$

$$(\langle 13,8,4 \rangle - \langle 15,8,2 \rangle^* + \langle 19,4,2 \rangle^*) \uparrow^{(9,3)} S_{26} \Rightarrow a_{12} \geq a_9. \text{ Hence } a_9 = a_{12}, \quad (23)$$

then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $Y_1 + Y_2 = n_1 k_8 + n_2 k_9$, such that $n_1 \in \{1,2,3,4\}$, $n_2 \in \{0,1, \dots, 6 - n_1\}$, so $k_8 = d_{19} + d_{20}$.

Also $\langle 15,8,3 \rangle \neq \langle 15,8,3 \rangle'$. Let $a_9 \in \{1, \dots, 6\}$. By restricting and inducing we get: $Y_1 = a_9 \langle 15,8,3 \rangle + a_{10} \langle 15,7,4 \rangle + a_{11} \langle 15,6,5 \rangle + a_{12} \langle 14,8,4 \rangle + a_{13} \langle 13,9,4 \rangle + a_{16} \langle 11,9,4,2 \rangle^* + a_{17} \langle 11,8,4,3 \rangle^*$, $Y_2 = a_9 \langle 15,8,3 \rangle' + a_{10} \langle 15,7,4 \rangle' + a_{11} \langle 15,6,5 \rangle' + a_{12} \langle 14,8,4 \rangle' + a_{13} \langle 13,9,4 \rangle' + a_{16} \langle 11,9,4,2 \rangle^* + a_{17} \langle 11,8,4,3 \rangle^*$ and since

$$(\langle 12,9,4 \rangle^* - \langle 11,9,4,1 \rangle + \langle 9,8,4,3,1 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{13} \geq a_{16} \quad (24)$$

$$(\langle 11,9,4,1 \rangle - \langle 12,9,4 \rangle^* + \langle 15,9,1 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{16} \geq a_{13}. \text{ Hence } a_{13} = a_{16} \quad (25)$$

$$(\langle 16,6,3 \rangle^* - \langle 14,6,5 \rangle^* + \langle 11,6,5,3 \rangle) \uparrow^{(4,8)} S_{26} \Rightarrow 0 \geq a_{11}. \text{ Hence } a_{11} = 0 \quad (26)$$

$$(\langle 10,9,4,2 \rangle - \langle 10,8,4,3 \rangle + \langle 10,6,5,4 \rangle) \uparrow^{(1,11)} S_{26} \Rightarrow a_{13} \geq a_{17} \quad (27)$$

$$(\langle 10,8,4,3 \rangle - \langle 10,9,4,2 \rangle^* + \langle 11,10,4 \rangle^*) \uparrow^{(1,11)} S_{26} \Rightarrow a_{17} \geq a_{13}. \quad (28)$$

$$\text{Hence } a_{13} = a_{17} \quad (28)$$

$$(\langle 14,8,3 \rangle^* + \langle 15,6,5 \rangle^* - \langle 14,7,4 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_9 \geq a_{10} \quad (29)$$

$$(\langle 14,7,4 \rangle^* + \langle 14,9,2 \rangle^* - \langle 14,8,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} \geq a_9. \text{ Hence } a_9 = a_{10}, \quad (30)$$

then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $Y_1 + Y_2 = n_1 k_9 + n_2 k_{11}$, $n_1 = 1, n_2 \in \{0,1, \dots, 4\}$ or $n_1 \in \{2,3, \dots, 5\}$, $n_2 \in \{0,1, \dots, 6 - n_1\}$. So $k_9 = d_{21} + d_{22}$.

Since $\langle 15,6,4 \rangle \neq \langle 15,6,4 \rangle'$, let $a_{11} = 1$. By restricting and inducing we get:

$$Y_1 = a_{10}\langle 15,7,4 \rangle + a_{11}\langle 15,6,5 \rangle + a_{17}\langle 11,8,4,3 \rangle^* + a_{18}\langle 11,6,5,4 \rangle^*, Y_2 = a_{10}\langle 15,7,4 \rangle' + a_{11}\langle 15,6,5 \rangle' + a_{12}\langle 14,8,4 \rangle' + a_{17}\langle 11,8,4,3 \rangle^* + a_{18}\langle 11,6,5,4 \rangle^* \text{ and, since}$$

$$(\langle 11,6,5,3 \rangle - \langle 11,7,4,3 \rangle + \langle 11,9,4,1 \rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{18} \geq a_{11} \quad (31)$$

We set $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $a_{10} = a_{12} = a_{17} = a_{18} = 1$. It follows that $k_{10} = d_{23} + d_{24}$.

For k_{11} let $a_{12} \in \{1, \dots, 6\}$. By restricting and inducing we get on and, since

$$(\langle 13,9,3 \rangle^* - \langle 14,8,3 \rangle^* + \langle 15,7,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{13} \geq a_{12} \quad (32)$$

$$(\langle 14,8,3 \rangle^* + \langle 12,10,3 \rangle^* - \langle 13,9,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{12} \geq a_{13}. \text{Hence } a_{12} = a_{13} \quad (33)$$

$$(\langle 12,9,4 \rangle^* - \langle 11,9,4,1 \rangle + \langle 9,8,4,3,1 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{12} \geq a_{16} \quad (34)$$

$$(\langle 11,9,4,1 \rangle - \langle 12,9,4 \rangle^* + \langle 15,9,1 \rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{16} \geq a_{12}. \text{Hence } a_{12} = a_{16}, \quad (35)$$

then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $Y_1 + Y_2 = nk_{11}$, $n \in \{1, 2, 3, 4\}$, so $k_{11} = d_{25} + d_{26}$.

For k_{12} let $a_{13} \in \{1, \dots, 5\}$. By restricting and inducing, we get

$$(\langle 11,10,3,1 \rangle - \langle 11,9,3,2 \rangle + \langle 10,9,3,2,1 \rangle^* + \langle 14,7,4 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{15} \geq a_{16} \quad (36)$$

$$(\langle 11,9,3,2 \rangle - \langle 11,10,3,1 \rangle + \langle 14,11 \rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{16} \geq a_{15}. \text{Hence } a_{15} = a_{16} \quad (37)$$

$$(\langle 11,10,3,1 \rangle - \langle 13,9,3 \rangle^* + \langle 14,8,3 \rangle^* + \langle 14,9,2 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{15} \geq a_{13} \quad (38)$$

$$(\langle 13,9,3 \rangle^* - \langle 11,10,3,1 \rangle + \langle 14,11 \rangle + \langle 10,9,3,2,1 \rangle^* + \langle 10,7,4,3,1 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{13} \geq a_{15}. \text{Hence } a_{13} = a_{15}. \quad (39)$$

Then $k_{12} = d_{27} + d_{28}$. For k_{13} let $a_{14} \in \{1, 2\}$. By restricting and inducing we get on:

$$(\langle 11,6,5,3 \rangle - \langle 11,7,4,3 \rangle + \langle 10,9,3,2,1 \rangle^* + \langle 13,9,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{17} = 0 \quad (40)$$

$$(\langle 11,10,3,1 \rangle - \langle 11,6,3,2 \rangle + \langle 11,7,4,3 \rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{15} \geq a_{16} \quad (41)$$

$$(\langle 11,9,3,2 \rangle - \langle 11,10,3,1 \rangle + \langle 14,11 \rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{16} \geq a_{15}. \text{Hence } a_{15} = a_{16} \quad (42)$$

$$(\langle 10,6,5,3,1 \rangle^* - \langle 9,6,5,3,2 \rangle^* + \langle 10,9,3,2,1 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{21} \geq a_{23} \quad (43)$$

$$(\langle 9,6,5,3,2 \rangle^* - \langle 10,6,5,3,1 \rangle^* + \langle 11,6,5,3 \rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{23} \geq a_{21}. \quad (44)$$

$$\text{Hence } a_{21} = a_{23},$$

then $\deg Y_1, Y_2 \equiv 0 \pmod{11^2}$ only when $Y_1 + Y_2 = n_1 k_{13} + n_2 k_{15}$, $n_1 = 0, n_2 = 1$ or $n_1 = 1, n_2 \in \{0, 1\}$. So $k_{13} = d_{31} + d_{32}$.

For k_{14} , let $a_{24} = 1$. By restricting and inducing then it splits to d_{37} and d_{38} . For k_{15} let $a_{23} \in \{1, 2\}$ and since

$$(\langle 10,9,3,2,1 \rangle^* - \langle 7,6,5,4,3 \rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{24} = 0. \quad (45)$$

So it's splits to d_{39} and d_{40} . Finally we have 206 columns, and then k_7 must divided into d_{17} and d_{18} . The decomposition matrix is shown in Table (3).

References

- [1] G. D. James, A. Kerber, The representation theory of the symmetric group. Mass, Addison-Wesley(1981)
- [2] A.O. Morris, A. K. Yaseen, Decomposition matrices for spin characters of symmetric group. Proc. of Royal society of Edinburgh 108A (1988)145-164.
- [3] M. M. Jawad, On Brauer Trees and Decomposition Matrices of Spin Characters of S_n Modulo 13,11(2018).

- [4] A.O. Morris, The spin representation of the symmetric group. *proc. London Math. Soc* (3)12 (1962) 55-76
- [5] B.M. Puttaswamaiah, J.D. Dixon, Modular representation of finite groups. Academic Press, *J.london Math. Soc* 15 (1977) 445-455.
- [6] A. H. Jassim: 7-Modular Character of The Covering group $\bar{S}_{23}$. *Journal of Basrah Researches ((Sciences))* V 43. N. 1 A (2017) 108-129.
- [7] I. Schur, Uber die Darstellung der symmetrischen und der alternierenden gruppe durch gebrochene lineare substituttionen. *j.Reine ang.Math* 139(1911) 155-250.
- [8] J. F. Humphreys, Projective modular representations of finite groups. *J. London Math. Society* 2.16(1977)51-66.

مصفوفات التجزئة للمشخصات الأسقاطية للزمرة التناظرية $p = 11, S_{26}$

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المستخلص

في هذا البحث لقد وجدنا مصفوفات التجزئة للمشخصات الأسقاطية للزمرة التناظرية $p = 11, S_{26}$ والتي هي علاقة بين المشخصات الاعتيادية والمعيارية